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Helicon solitons in a layered semiconductor plasma via Zakharov equations

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Abstract. In the present work we investigate the propagation of helicon envelope solitons in a layered semiconductor plasma. The nonlinear evolution equations governing the propagation of these envelope solitons is the set of Zakharov equations (which are a more generalized form of the nonlinear Schrödinger equation). The set of equations which have a known envelope soliton solution are derived and the relationship between various parameters entering the system is established. In order to investigate the propagation of helicon envelope solitons in a layered medium we use the standard Kronig–Penney model along with its relevant boundary conditions. These boundary conditions are used for the envelope soliton solution thereby connecting the envelope soliton fields across the layers. This in turn leads to a nonlinear dispersion relation which relates the nonlinear analogue of the Bloch wave number with different parameters. We have numerically investigated the dependence of the nonlinear Bloch wave number on the propagation frequency and have established a propagation band and gap structure for the helicon envelope soliton in a layered semiconductor plasma.

1. Introduction

Helicons are transverse circularly polarized waves, which propagate parallel to the ambient magnetic field in plasmas and plasma-like media i.e. semiconductors, conducting layered media and superlattices. In the last quarter of a century the linear theory of helicons in layered media and superlattices has received considerable attention, beginning with the work of Baynham and Boardman (1968, 1969) which is considered a watershed for subsequent development which took place in this area. In a later paper Baynham and Boardman (1970) reviewed the progress on helicon and Alfven wave propagation in semiconductors and semimetals. This work provides a comprehensive description of the above-mentioned waves in different (bounded and unbounded) semiconductor and semimetal plasmas including layered media. The existence of helicon waves was suggested in the experimental work of Mann *et al* (1982), whilst theoretical research in this field can be found in many works (Tselis *et al* 1983, Tselis and Quinn 1984, Kushwaha 1986, Sarma and Quinn 1982). Non-local effects in helicon wave propagation have been considered by Achar (1987a). In most of these papers the layered medium or superlattice has been modelled in a Kronig–Penney type of periodic structure. Achar (1987b) has considered, as an alternative to the Kronig–Penney model, a sinusoidally modulated periodic structure.

The references given above limit themselves to linear investigations. However there has been considerable progress in nonlinear theory of wave propagation and wave interaction

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in layered medium and superlattices (see for example Bass and Tetervov 1986, 1988 and references therein). However all the investigations referred to by Bass and Tetervov use an effective medium approach rather than a Kronig–Penney model for the superlattices. More recently Shah *et al* (1993) used a Kronig–Penney model to investigate helicon soliton propagation in a layered medium. The equation governing the propagation of the helicon solitons was found to be a mixture of the Korteweg–de Vries (KdV) equation and the nonlinear Schrödinger (NLS) equation (Shah *et al* 1993), where a reductive perturbation technique was used to arrive at the above-mentioned equation. Later Rashid *et al* (1995) used Achar's model (Achar 1987b) of a sinusoidal periodic structure to investigate the parametric interaction of waves in a layered medium.

In the present work we undertake to examine the helicon wave solitons via Zakharov equations (Zakharov 1972). Zakharov equations are a more generalized form of the nonlinear Schrödinger equation referred to above. These equations can be solved self-consistently and yield envelope soliton solutions where the wave under consideration can be found to be modulationally unstable. We have used the Kronig–Penney model to describe the layered semiconducting medium, and by using a scaling based on physical considerations of the helicon mode, a set of Zakharov equations is derived. These equations are solved and a nonlinear dispersion relation relating different parameters of the layered structure is obtained. Boundary conditions of the Kronig–Penney model are used for the envelope soliton solution, thereby relating different quantities in different layers.

2. Mathematical formulation

In order to study the propagation of helicons in a layered semiconductor medium, the following fundamental set of equations is used

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial z}(nv_z) = 0 \tag{1}$$

$$\frac{\partial v_\pm}{\partial t} + v_z \frac{\partial}{\partial z} v_\pm = -\frac{e}{m} E_\pm \pm \mathrm{i}\omega_c v_\pm \mp \mathrm{i}\frac{e}{m} v_z B_\pm \tag{2}$$

$$\frac{\partial v_z}{\partial t} + v_z \frac{\partial}{\partial z} v_z = -\frac{e}{m}(v_x B_y - v_y B_x) - \frac{V_T^2}{n}\frac{\partial}{\partial z} n \tag{3}$$

$$\frac{\partial B_\pm}{\partial z} = \mp \mathrm{i}\mu j_\pm \mp \mathrm{i}\epsilon\mu \frac{\partial}{\partial t} E_\pm \tag{4}$$

$$\frac{\partial}{\partial z} E_\pm = \pm \mathrm{i} \frac{\partial}{\partial t} B_\pm \tag{5}$$

and

$$J_\pm = -nev_\pm. \tag{6}$$

In the above equations n, v_z, $v_\pm$ and $j_\pm$ denote the electronic number density, the parallel velocity, the perpendicular velocities (with respect to the background magnetic field) and the perpendicular current densities respectively. $E_\pm$ and $B_\pm$ are fluctuating perpendicular electric and magnetic fields respectively. The quantities μ, ε, ω_c and v_T are the magnetic susceptibility, the dielectric constant, the electron cyclotron frequency and electron thermal velocity respectively. Since helicons are circularly polarized waves the perpendicular fluctuating quantities have all been expressed in the form

$$a_\pm = a_x \pm \mathrm{i}a_y. \tag{7}$$

The background magnetic field is directed normal to the semiconducting layers and is parallel to the z-axis. We note here that in the above equations the various parameters have different values in different layers. We would also like to state that we are using a semiclassical fluid model (1)–(6) and this is valid for layered media as long as the thickness of each layer is greater than the wavelength of the wave under consideration.

Substituting (4) and (6) in (2) and eliminating $E_\pm$ and $v_\pm$ by using (5), we arrive at the following equation for $B_\pm$

$$\left[\frac{\partial}{\partial t}\frac{\partial^2}{\partial z^2} + v_{z1}\frac{\partial^3}{\partial z^3} - \frac{1}{c^2}\frac{\partial^3}{\partial t^3} - \frac{v_{z1}}{c^2}\frac{\partial^2}{\partial t^2}\frac{\partial}{\partial z} - \frac{\omega_p^2}{c^2}\frac{n_1}{n_0}\frac{\partial}{\partial t} - \frac{\omega_p^2}{c^2}v_{z1}\frac{\partial}{\partial z} - \frac{\omega_p^2}{c^2}\frac{\partial}{\partial t} \mp \mathrm{i}\omega_c\frac{\partial^2}{\partial z^2} \pm \mathrm{i}\frac{\omega_c}{c^2}\frac{\partial^2}{\partial t^2}\right]B_\pm = 0. \tag{8}$$

We note that in obtaining (8), we have included only first-order contributions from the electron number density and parallel velocity (see below and the scaling procedure in section 3).

In order to proceed with the linear analysis we note that for helicon waves, number density fluctuations and fluctuations in the parallel electronic velocity are absent, therefore contributions from these terms are important only when nonlinearities in the magnetic field fluctuations are taken into account. Thus by putting

$$n = n_0$$

and

$$v_{z1} = 0$$

(8) reduces to the linear differential equation in $B_\pm$ and in order to obtain the linear dispersion relation for helicon waves we can use a plane wave solution of the form

$$B_\pm \sim \exp\left[\pm\mathrm{i}(k_\xi + k_\eta)z \mp \mathrm{i}\omega t\right].$$

This yields the same result as that of Baynham and Boardman (1968, 1969) in their collisionless limit, and k_ξ and k_η are the same as k and (α, β) of Baynham and Boardman (1968, 1969). When the periodic structure is replaced by an infinite semiconductor, k_η vanishes.

3. Zakharov equations

In order to obtain the set of Zakharov equations, we begin by considering (8) and we follow the scaling scheme used by Ovenden *et al* (1983) who investigated the propagation of Alfven solitons in the solar wind. Thus in order to obtain the first Zakharov equation, we proceed in the following manner. Terms containing n and v_z are now retained to first order—this is equivalent to a multiple-scale analysis where perturbations in n and v_z are of the order of the square of perturbations in the fluctuating magnetic field.

$$B = b(z, t)\exp[\mathrm{i}(k_\xi + k_\eta)z - \omega t]$$

where now the amplitude b is also a function (slowly varying) of the variables z and t.

Following Ovenden *et al* (1983) we introduce the following scaling procedure

$$\frac{\partial b}{\partial t} \sim \frac{b}{\tau} \qquad \frac{\partial}{\partial z}b \sim \frac{b}{V_g\tau} \qquad v_{z1} \sim V_g b^2$$

and

$$n_1 \sim b^2$$

where τ is a time scale and $1/\tau < \omega$; ω is of the order of the helicon wave propagation frequency.

Using the above-mentioned scaling, we obtain the following expression to order b^3 and $1/\tau$.

$$\mathrm{i}\frac{\partial b}{\partial t} + \mathrm{i}V_g\frac{\partial}{\partial z}b + \frac{1}{2}\frac{\partial V_g}{\partial(k_\xi + k_\eta)}\frac{\partial^2}{\partial z^2}b + \frac{v_{z1}(k_\xi + k_\eta)[\pm\omega_p^2\omega_c/(\omega \pm \omega_c)] - \omega\omega_p^2(n_1/n_0)}{(\omega \pm \omega_c)[2\omega \mp \omega_p^2\omega_c/(\omega \pm \omega_c)^2]} = 0 \tag{9}$$

where

$$V_g = \frac{2c^2(k_\xi + k_\eta)}{[2\omega \mp \omega_p^2\omega_c/(\omega \pm \omega_c)^2]}. \tag{10}$$

(9) is the first Zakharov equation.

We note that v_g is the group velocity of the helicon wave obtained from the linear dispersion relation within each layer which is given by

$$c^2(k_\xi + k_\eta)^2 - \omega^2 = -\frac{\omega\omega_p^2}{\omega \pm \omega_c}. \tag{11}$$

The next Zakharov equation is obtained from the parallel equation of motion (3) and the continuity equation given by (1). By eliminating v_{z1}, the perpendicular velocity and magnetic field fluctuation we get

$$\left[\frac{\partial^2}{\partial t^2} - V_T^2\frac{\partial^2}{\partial z^2}\right]n_1 = \frac{1}{2m\mu}\frac{\partial^2}{\partial z^2}|b|^2 + \frac{\omega}{2m\mu(k_\xi + k_\eta)c^2}\frac{\partial}{\partial z}\frac{\partial}{\partial t}|b|^2. \tag{12}$$

The third Zakharov equation is the linearized continuity equation and is given by

$$\frac{\partial n_1}{\partial t} + n_0\frac{\partial}{\partial z}v_{z1} = 0. \tag{13}$$

(9), (12) and (13) are the set of Zakharov equations which have known envelope soliton solutions. We note here that if in (12) and (13) the time dependence of n_1 and $|b|^2$ is neglected then the set of (9), (12) and (13) reduces to the standard NLS equation. Thus we substitute in (9), (12) and (13) a solution of the form (Shah *et al* 1993)

$$b(z, t) = b_0\,\mathrm{sech}((\chi_\xi + \chi_\eta)z - \omega^* t)\,\mathrm{e}^{\mathrm{i}\delta\Omega t} \tag{14}$$

and solve in a self-consistent manner. We note here that b_0 is the maximum envelope soliton amplitude, χ_ξ and χ_η are the nonlinear wave numbers and are analogous to k_ξ and k_η respectively, ω^* is the corresponding nonlinear frequency and $\delta\Omega$ is the nonlinear frequency shift. We note that the solution given by the expression (14) refers to a frame of reference moving with the group velocity (see (16) below).

Thus we obtain the following results from (9)–(14)

$$\delta\Omega = \frac{1}{2}\frac{\partial V_g}{\partial(k_\xi + k_\eta)}(\chi_\xi + \chi_\eta)^2 \tag{15}$$

$$\omega^* = V_g(\chi_\xi + \chi_\eta) \tag{16}$$

$$(\chi_\xi + \chi_\eta)^2 = b_0^2 Y_1 \tag{17}$$

where

$$Y_1 = \frac{\omega_p^2}{4c^2 n_0\mu m(\omega + \omega_c)(V_g^2 - V_T^2)}\left[1 - \frac{\omega V_g}{(k_\xi + k_\eta)c^2}\right]\left[\frac{\pm\omega_c(k_\xi + k_\eta)V_g}{(\omega \pm \omega_c)} - \omega\right]. \tag{18}$$

We note here that if $\chi_\eta = 0$, then we obtain the case of the helicon soliton in an infinite semiconductor.

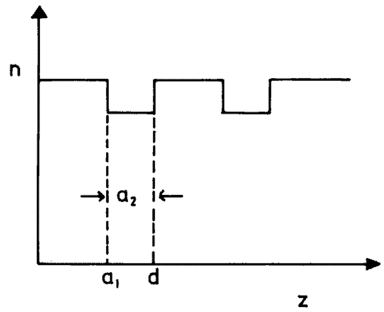


Figure 1. Periodic modulation of the number density.

4. Periodic boundary conditions

Since we are investigating the propagation of helicon solitons in a semiconducting layered medium consisting of two alternating semiconductor layers of thickness a_1 and a_2 respectively. These two layers repeat periodically (see figure 1), thus the soliton solution used in the previous section along with the expressions (15)–(18) that we obtained should carry a subscript i (where $i = 1, 2$) denoting different layers, for example expression (14) should be

$$b = b_{0i}\,\mathrm{sech}((\chi_{\xi i} + \chi_{\eta i})z - \omega_i^* t)\,\mathrm{e}^{\mathrm{i}\delta\Omega_i t}. \tag{19}$$

We now introduce the boundary conditions, which are used in the standard treatment of layered media having a Kronig–Penney structure, and wave propagation across the layers is considered (see for example Baynham and Boardman 1968, 1969). We note here that in the linear case (Baynham and Boardman 1968, 1969) the solution in a periodic medium is considered to be a superposition of transmitted and reflected wave in each layer, whereas in the nonlinear regime the envelope soliton solution (19) is the total field. We demand that the envelope soliton magnetic fields of the two layers are connected to one another at the boundary of the two layers in the following way

$$b_1|_{z=a_1} = b_2|_{z=a_1} \tag{20}$$

$$\left.\frac{\partial b_1}{\partial z}\right|_{z=a_1} = \left.\frac{\partial b_2}{\partial z}\right|_{z=a_1} \tag{21}$$

$$b_1|_{z=0} = \mathrm{e}^{\mathrm{i}\bar{K}d} b_2|_{z=d} \tag{22}$$

$$\left.\frac{\partial b_1}{\partial z}\right|_{z=0} = \mathrm{e}^{\mathrm{i}\bar{K}d}\left.\frac{\partial b_2}{\partial z}\right|_{z=d} \tag{23}$$

where $a_1 + a_2 = d$ and $\bar{K}$ is the nonlinear analogue of the Bloch wave number. We now substitute the solution for b_0 (given by (19)) into the set of (20)–(23). By skipping the details of the rather messy algebra for the nonlinear analogue of the Bloch wave number, we directly give the final expression that is obtained

$$W_1 \cos^2 \bar{K}d + W_2 \cos \bar{K}d + W_3 = 0 \tag{24}$$

where W_1, W_2 and W_3 are given by

$$\begin{aligned} W_1 &= X_1 C_1^2 C_2^2 - X_2 C_1^2 C_2 S_2 + X_3 C_1^2 S_2^2 \\ W_2 &= X_1 A_2 + X_2 D_2 + X_3 B_2 \\ W_3 &= X_1 A_1 + X_2 D_1 + X_3 B_1 \end{aligned} \tag{25}$$

and A_1, A_2, B_1, B_2 and D_1, D_2 are given by

$$\begin{aligned}
A_1 = {} & C_{11}^2C_1^2C_{22}^2C_2^2 + C_{11}^2C_1^2S_{22}^2S_2^2 + S_{11}^2S_1^2C_{22}^2C_2^2 + S_{11}^2S_1^2S_{22}^2S_2^2 + 2C_{11}^2C_1^2C_{22}C_2S_{22}S_2 \\
& +2C_{11}C_1C_{22}^2C_2^2S_{11}S_1 + 2C_{11}C_1S_{22}^2S_2^2S_{11}S_1 + 2S_{11}^2S_1^2C_{22}C_2S_{22}S_2 \\
& +4C_{11}C_1C_{22}C_2S_{11}S_1S_{22}S_2 \\
A_2 = {} & -C_{11}C_{22}C_1^2C_2^2 - 2C_1^2C_2C_{11}S_{22}S_2 - 2C_1C_2^2S_{11}S_1C_{22} - 2C_1C_2S_{11}S_1S_{22}S_2 \\
B_1 = {} & 2C_{11}^2C_1^2S_{22}C_2C_{22}S_2 + 2C_{11}C_1S_{22}^2C_2^2S_{11}S_1 + 2C_{11}C_1C_{22}^2S_2^2S_{11}S_1 + 2S_{11}^2S_1^2S_{22}C_2C_{22}S_2 \\
& -4C_{11}C_1S_{22}C_2S_{11}S_1C_{22}S_2 + C_{11}^2C_1^2S_{22}^2C_2^2 + C_{11}^2C_1^2C_{22}^2S_2^2 + S_{11}^2S_1^2S_{22}^2C_2^2 \\
& +S_{11}^2S_1^2C_{22}^2S_2^2 \\
B_2 = {} & -2C_1^2S_2C_{11}S_{22}C_2 - 2C_1^2S_2^2C_{11}C_{22} - 2C_1S_2S_{11}S_1S_{22}C_2 - 2C_1S_2^2S_{11}S_1C_{22} \\
D_1 = {} & -C_{11}^2C_1^2C_{22}C_2^2S_{22} - C_{11}^2C_1^2C_{22}^2C_2S_2 - 2C_{11}C_1C_{22}C_2^2S_{11}S_1S_{22} - 2C_{11}C_1C_{22}^2C_2S_{11}S_1S_2 \\
& -C_{11}^2C_1^2S_{22}^2S_2C_2 - C_{11}^2C_1^2S_2^2S_{22}C_{22} - 2C_{11}C_1S_{22}^2S_2S_{11}S_1C_2 \\
& -2C_{11}C_1S_{22}S_2^2S_{11}S_1C_{22} - S_{11}^2S_1^2C_2^2C_{22}S_{22} - S_{11}^2S_1^2C_{22}^2C_2S_2 \\
& -S_{11}^2S_1^2S_{22}^2S_2C_2 - S_{11}^2S_1^2S_{22}S_2^2C_{22} \\
D_2 = {} & 2C_1^2C_2C_{11}C_{22}S_2 + C_1^2C_2^2C_{11}S_{22} + C_1C_2^2S_{11}S_1S_{22} + 2C_1C_2S_{11}S_1C_{22}S_2 + C_{11}C_1^2S_{22}S_2^2 \\
& +S_{11}S_1S_{22}S_2^2C_1.
\end{aligned} \tag{26}$$

X_1, X_2 and X_3 are given by

$$\begin{aligned}
X_1 = {} & -S_1C_2^2C_{11}C_1S_{22} - S_1^2C_2^2S_{11}S_{22} - S_1^2C_2S_{11}C_{22}S_2 + C_1^2S_2S_{11}C_{22}C_2 \\
& +V_1^2S_2^2S_{11}S_{22} + C_1S_2^2C_{11}S_1S_{22} \\
X_2 = {} & -S_1^2C_2^2S_{11}C_{22} - 2S_1^2S_2S_{11}S_{22}C_2 - S_1^2S_2^2S_{11}C_{22} + C_1^2C_2^2S_{11}C_{22} \\
& +2C_1^2S_2S_{11}S_{22}C_2 + C_1^2S_2^2S_{11}C_{22} \\
X_3 = {} & -S_1S_2^2C_{11}S_{22}C_1 - S_1^2S_2S_{11}C_{22}C_2 - S_1^2S_2^2S_{11}S_{22} + C_1^2S_{11}S_{22} \\
& +C_1^2C_2S_{11}C_{22}S_2 + C_1C_2^2C_{11}S_1S_{22}
\end{aligned} \tag{27}$$

where S_1, S_2, C_1, C_2, S_{11}, S_{22}, C_{11}, C_{22} and S_{21}, C_{21} are given by

$$\begin{aligned}
& S_1 = \sinh\Theta_1 \qquad S_{11} = \sinh\chi_1a_1 \qquad S_{21} = \sinh\chi_2a_1 \\
& S_2 = \sinh\Theta_2 \qquad S_{22} = \sinh\chi_2a_2 \qquad C_{21} = \cosh\chi_2a_1 \\
& C_1 = \cosh\Theta_1 \qquad C_{11} = \cosh\chi_1a_1 \\
& C_2 = \cosh\Theta_2 \qquad C_{22} = \cosh\chi_2a_2.
\end{aligned} \tag{28}$$

Here

$$\Theta_i = \omega_i^*t \qquad \chi_i = \chi_{\xi i} + \chi_{\eta i}$$

where

$$i = 1, 2$$

S_{21} and C_{21} do not appear explicitly in expressions (25)–(27) as these have been eliminated and expressed in terms of hyperbolic functions S_{ii} and C_{ii}.

Equation (24) is the nonlinear dispersion relation for helicon envelope solitons propagating across the layers of two alternating semiconductors. We note here that (24) is quadratic in $\cos(\bar{K}d)$, whereas in the linear case (Baynham and Boardman 1968) the dispersion relation was linear in $\cos(\bar{K}d)$. Thus the helicon soliton has two modes of propagation corresponding to the two solutions of (24); we further see that this is a complicated equation and $\bar{K}d$ depends on the different parameters entering the system in a complex fashion (see expressions (25)–(28)).

5. Numerical analysis and conclusions

In this section we numerically investigate (24) by taking some numerical values associated with typical layered media and we attempt to establish a relationship between the nonlinear analogue of the Bloch wave number $\bar{K}$ and the propagation frequency ω. As pointed out in the preceding section (24) has two solutions (as opposed to the linear case (Baynham and Boardman 1968) where only one solution is obtained). These solutions are given by

$$\cos \bar{K} d = \frac{-W_2 \pm \sqrt{W_2^2 - 4W_1 W_3}}{2W_1} \tag{29}$$

and W_1, W_2 and W_3 are defined by expressions (25)–(28). We note that real propagating roots will be obtained only when

$$|\cos \bar{K} d| \leqslant 1. \tag{30}$$

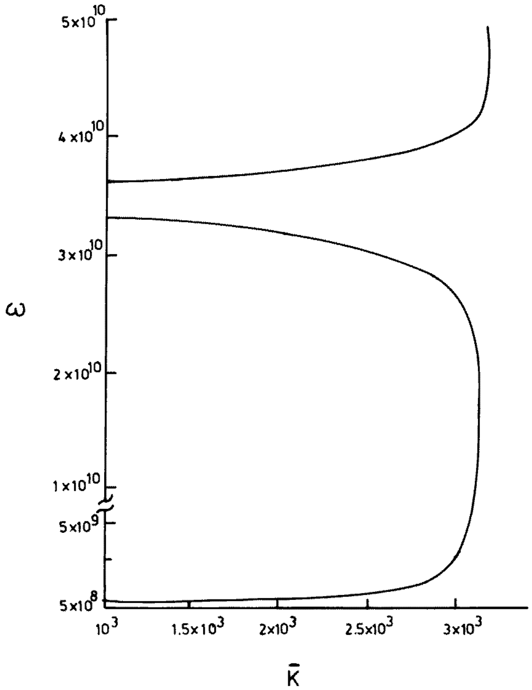
The numerical investigations that we have carried out are for a metal–semiconductor sandwich system which was used for the linear regime (Baynham and Boardman 1968). Numerical values of the linear wave number $k_{\xi 1,2}$ are calculated from the linear dispersion relation (11) by using the fact that for the linear wave vector across the layer, $k_{\eta 1,2}$ is of the order of $1/a_{1,2}$ and we further note that the nonlinear frequency ω^* and nonlinear wave vector $\chi_{\xi 1,2}$ within the layer are taken as $\omega^* \ll \omega$; $\chi_{\xi 1,2} \ll k_{\xi 1,2}$ and $\chi_{\eta 1,2}$ is then obtained through (16) which is subsequently used in (29).

Figure 2 shows the dependence of the nonlinear Bloch wave number $\bar{K}$ on the propagation frequency ω, for two different values of the magnetic field B_0 (1 T, 10 T respectively). The numerical values used for the thicknesses a_1, a_2 and the dielectric constants ε_1, ε_2; the number densities n_1, n_2; the effective masses m_1, m_2 and values of Θ_1, Θ_2, are given in the captions to figure 2. Comparisons of figure 2(a) and (b) show that in each case there is a propagation gap and propagation bands; however as the magnetic field increases, the frequency increases, thus both propagation bands and gaps shift to the right. This is qualitatively similar to the case investigated by Baynham and Boardman (1968). However in the nonlinear regime we see that as ω increases $\bar{K}$ increases and then decreases, giving us a region of propagation after which there is a propagation gap and then again there is a region of propagation. We also note from figure 2 that the propagation gap widens as $\bar{K}$ increases in magnitude. This is opposite to the behaviour of the propagation gap as described by Baynham and Boardman (1968). Thus investigation of the nonlinear regime via Zakharov equations shows that the linear effect is offset, although the band-gap structure in the propagation characteristics of the helicon wave in a layered medium is maintained.

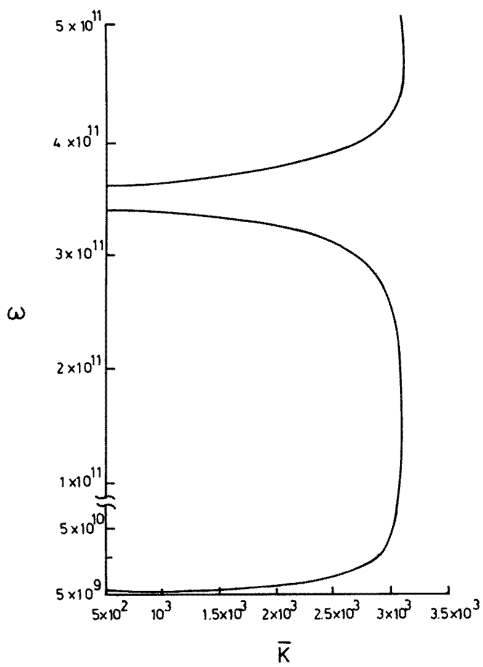
We note here that numerical investigations presented above are for the root with the upper sign in (29). For the type of numerical value that we have used the negative sign gives values which do not comply with the condition given by expression (30). Thus the lower sign in the aformentioned expression results in non-physical solutions.

Further we would like to mention that we have presented a numerical example of a metal–semiconductor layered medium in order to make comparison with the linear results of Baynham and Boardman (1969), and did not take into account the effect of the Schottky barrier, which is formed in metal–semiconductor contacts. However the effect of the Schottky barrier could be included by modifying the current term in the set of equations (1)–(6) for the semiconductor plasma by using the 'depletion approximation' (see, for example, Rhoderick and Williams 1988) which would entail the case of a quasi-Fermi level for electrons. This is beyond the scope of the present paper, but could form the basis for subsequent investigations.

Our investigations show that for nonlinear helicon waves having soliton solutions, the band gap structure in the propagation characteristics is maintained when a Kronig–Penney model is used to depict a sandwich structure.



(a)



(b)

Figure 2. Nonlinear analogue of the Bloch wave number $\bar{K}$ versus helicon wave frequency ω for magnetic fields (a) $B = 1$ T (b) $B = 10$ T. Other parameters are $a_1 = 5 \times 10^{-4}$ m, $a_2 = 10^{-5}$ m; $m_1 = 0.1m_0$, $m_2 = 1.0m_0$; $\varepsilon_1 = 10$, $\varepsilon_2 = 1$; $n_1 = 10^{23}$ m^{-3}, $n_2 = 10^{29}$ m^{-3} and $\Theta_1 = 10^{-3}$, $\Theta_2 = 10^{-4}$. The scale is changed at the $\approx$ mark.

We had set out to describe nonlinear wave propagation of helicon waves, in a layered medium via the set of Zakharov equations. This set has been derived and by using a Kronig–Penney model with appropriate boundary conditions, we have derived relationships for the propagation of the helicon solitons in a layered medium. Further a nonlinear dispersion relation, relating the nonlinear Bloch wave number $\bar{K}$ with the propagation frequency ω has been derived. We note that the dependence of ω upon $\bar{K}$ is a complicated one due to the presence of the factors W_1, W_2 and W_3, see (29). This is resolved by carrying out a numerical analysis, the results of which have been presented graphically.

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